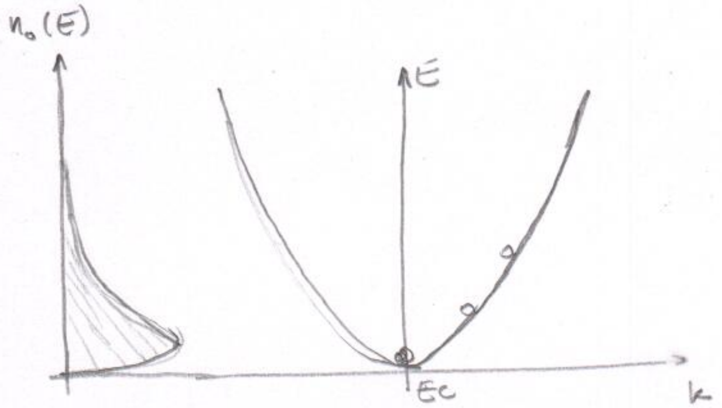
Course 8

thermal velocity



electrons with high E have higher velocity

Electron velocity:

$$v_n(k) = \frac{1}{\hbar} \nabla_k E_n(k)$$

at bottom of the conduction band:

$$E = \frac{\hbar^2 k^2}{2m^*} = \frac{p^2}{2m^*} \longrightarrow p\text{: momentum}$$

$$\text{thus} \quad v_n(k) = \frac{\hbar k}{m^*} = \frac{p}{m^*} \quad \Rightarrow p = \hbar k$$

classically: $p = mv$

$$v_e(E) = \sqrt{\frac{2(E-E_c)}{m^*_{ce}}}$$

m^*_{ce} → electron conductivity effective mass ⇕ connects kinetic energy with electron velocity

Average thermal velocity

$$\langle v_{th} \rangle = \frac{\int_{E_c}^{\infty} v_e(E)\, n_o(E)\, dE}{\int_{E_c}^{\infty} n_o(E)\, dE}$$

: average of all electron velocities at E above Ec

$$\langle v_{th} \rangle = \sqrt{\frac{2}{m^*_{ce}}} \cdot \frac{\int_{E_c}^{\infty} \frac{(E-E_c)}{1+\exp\frac{E-E_F}{kT}}\, dE}{\int_{E_c}^{\infty} \frac{\sqrt{E-E_c}}{1+\exp\frac{E-E_F}{kT}}\, dE} \overset{*}{=} \sqrt{\frac{8kT}{\pi m^*_{ce}}}$$

* derivation is on moodle, in the book chapter

for Si at Room temperature

$$m^*_{ce} = 0.28\, m_o \quad \Rightarrow \quad \langle v_e^{th} \rangle = 2 \times 10^7 \text{ cm/s}$$

Mean free path (thermal)

$$l_{ce} = \frac{\pi}{4} \cdot v_{th} \cdot \tau_{ce}$$: energy independent since

time between collisions

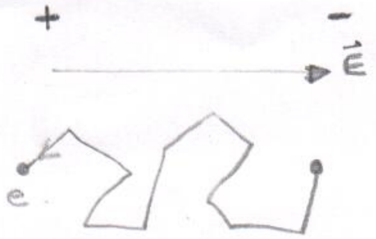
$v_{th} \propto (E-E_C)^{1/2}$

$\tau \propto (E-E_C)^{-1/2}$

$\tau_{ce} \propto (E-E_C)^{-1/2}$

Drift :

Under $\vec{E}$:



$\vec{F} = -q\,\vec{E}$

$\vec{a} = -\dfrac{q\,\vec{E}}{m^*_{ce}}$

$\vec{v}_{drift} = -\dfrac{q\,\vec{E}}{m^*_{ce}} \cdot \tau_{ce} = - \mu_e \cdot \vec{E}$

$\hookrightarrow$ electrons move against $\vec{E}$

$$\begin{cases} v_e^{drift} = -\mu_e \, E \\ v_h^{drift} = +\mu_h \cdot E \end{cases}$$

. we can define an electron mobility $[cm^2/V.s]$

Depends on strength of scattering mechanisms

Doping $\uparrow$ scattering $\uparrow$ $\mu \downarrow$ $v_{drift} \downarrow$

Mobility depends on :

- $1/T$
- $1/m^*$
- $1/\text{scattering}$ $\longrightarrow$ ionized impurities
  - acoustic phonons
  - optical phonons

* Example (page 3)

#12

Drift current :

$F_e\, dt = n.\, v_e\, dt \Rightarrow F_e = n\, v_e \quad [cm^{-3} . \frac{cm}{s}]$

$J_e = -q \cdot n\, v_e = -q.\, n.\, (-\mu_e\, E) = q.\, n.\, \mu_e . E$

$J_h = +q \cdot p \cdot v_h = q \cdot p\, \mu_h \cdot E$

$\boxed{J = q\,(n\mu_e + p\mu_h) \cdot E}$ ohm's law for semiconductors.

$V = R.I \Rightarrow \frac{V}{cm} = (R.cm)\frac{I}{cm^2} \Rightarrow E = \rho \cdot J$

Consider: a n-type Si with a resistivity of 0.1 Ω cm.

an applied electric field of 1000 V/cm

- what is the electron and hole drift velocity?
- what is the total drift current density?

0.1 Ω cm $\Rightarrow$ $N_D \approx 9\times10^{16}\ \text{cm}^{-3}$ $\qquad$ $n_i = 10^{10}\ \text{cm}^{-3}$

From the figure

Assuming $n_0 \simeq N_D$ $\Rightarrow$ $p_0 = \dfrac{(1\times10^{10})^2}{9\times10^{16}} = 1.1\times10^{3}\ \text{cm}^{-3}$

From the second plot:

$$\mu_e = 750\ \text{cm}^2/\text{Vs}$$

$$\mu_h = 460\ \text{cm}^2/\text{Vs}$$

thus:

$$v_e^{drift} = -\mu_e\, \mathcal{E} = -750\ \text{cm}^2/\text{Vs} \times 1000\ \text{V/cm} = -7.5\times10^{5}\ \text{cm/s}$$

$$v_h^{drift} = +\mu_h\, \mathcal{E} = 460\ \text{cm}^2/\text{Vs} \times 1000\ \text{V/cm} = 4.6\times10^{5}\ \text{cm/s}$$

much smaller than v_{th}
$\Rightarrow$ not in saturation.

Drift current densities:

$$J_e = -q\,n\,v_e^{drift} = +1.6\times10^{-19}\,\text{C} \times 9\times10^{16}\ \text{cm}^{-3} \times 7.5\times10^{5}\ \text{cm/s} = 1.1\times10^{4}\ \text{A/cm}^2$$

$$J_h = q\,p\,v_h^{drift} = 1.6\times10^{-19}\,\text{C} \times 1.3\times10^{3}\ \text{cm}^{-3} \times 4.6\times10^{5}\ \text{cm/s} = 9.6\times10^{-11}\ \text{A/cm}^2$$

very small since these are minority carriers.

$$\therefore\ J = J_e + J_h \simeq 1.2\times10^{4}\ \text{A/cm}^2$$

Course 8 -

. Energy band diagram under Electric field

. Potential Energy : $E_P = -q \cdot \phi$

. electrons go down the energy potential profile

⇒ trade E_P by Kinetic energy.

⇒ balance between phonon emission and absorption is <u>broken</u>.

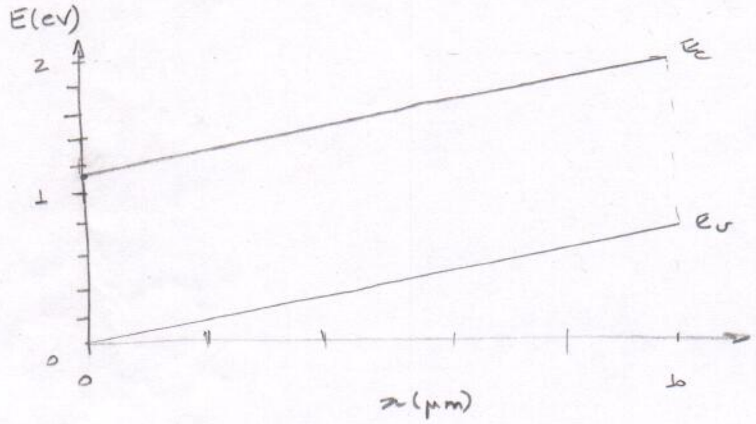
. more likely to <u>emit</u> phonons than absorb.

⇒ path to restore T.E.

#2

$$\begin{cases} E_c + E_{ref} = E_P = -q \cdot \phi \\ \vec{\mathcal{E}} = -\dfrac{d\phi}{dx} = \dfrac{1}{q} \cdot \dfrac{dE_c}{dx} = \dfrac{1}{q} \dfrac{dE_v}{dx} \end{cases}$$

<u>Example</u>: what is the magnitude and direction of $\vec{\mathcal{E}}$



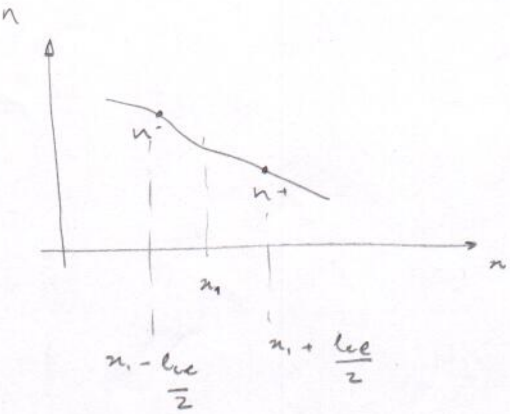
$$|\vec{\mathcal{E}}| = \frac{1}{q} \cdot \frac{dE_c}{dx} = \frac{1}{e} \cdot \frac{2 - 1.1}{10 \times 10^{-4}\,cm} = 900\ V/cm$$

$$\overset{+}{}\xrightarrow[\vec{\mathcal{E}}]{}\overset{-}{}$$

Diffusion :

Very common process : . drop of cream in coffee
.

- $\bar{e}$ and holes get kicked out by vibrating atoms. (at a certain T)
- at $T = 0$ $\Rightarrow$ no diffusion



flux :

$$F_e(x_1) = \frac{n^-}{2} \cdot v_{the} - \frac{n^+}{2} \cdot v_{the} = -\frac{1}{2} v_{the} (n^+ - n^-)$$

$$n^+ - n^- = \left.\frac{dn}{dx}\right|_{x_1} \cdot l_{ce}$$

$$l_{ce} = \frac{\pi}{4} v_{th} \cdot \tau_{ce}$$

$$\therefore F_e(x_1) = \underbrace{-\frac{1}{2} v_{the} \cdot l_{ce}}_{\substack{D_e \\ \text{diffusion coefficient}}} \cdot \left.\frac{dn}{dx}\right|_{x = x_1}$$

$$\boxed{D_e = \frac{1}{2} \cdot v_{the} \cdot l_{ce}}$$

$l_{ce} = \tau_{ce} \cdot \frac{\pi}{4} \cdot v_{th}$

and
$$\begin{cases} D_e = \frac{\pi}{8} \cdot v_{th}^2 \cdot \tau_{ce} \\ \mu = \frac{q \cdot \cancel{\ }}{m^*_{ce}} \cdot \tau_{ce} \end{cases}$$

$$v_{th} = \sqrt{\frac{8kT}{\pi m^*_{ce}}}$$

$$\Rightarrow \frac{D_e}{\mu} = \frac{\pi}{8} \cdot \frac{\cancel{\ }}{q} \cdot \frac{8kT}{\pi m^*_{ce}} = kT/q$$

$$\begin{cases} F_e = -D_e \cdot \frac{dn}{dx} \\ F_h = -D_h \cdot \frac{dp}{dx} \end{cases}$$

$\therefore$ flux is opposite to $\frac{dn}{dx}$

$\uparrow D$ $\uparrow$ diffusion

- . strength of scattering mechanisms
- . the longer l_{ce}, the higher the diffusion flux

Einstein relations

$$\begin{cases} \frac{D_e}{\mu_e} = \frac{kT}{q} \\ \frac{D_h}{\mu_h} = \frac{kT}{q} \sim V \end{cases}$$

$[cm^2/s]$

$[cm^2/Vs]$

$\therefore$ D, μ are intimately related to scattering mechanisms.

Exercise

- Estimate electron and hole diffusion coefficients in:

$$\begin{cases} \text{p-type Si at RT} \\ \text{resistivity of } 0.1\ \Omega\,cm \end{cases} \qquad D_h = \frac{KT}{q} \cdot \mu_h$$

1) Doping level $\Rightarrow \mu_h$

$$0.1\ \Omega\,cm \Rightarrow N_A \simeq 3 \times 10^{17}\ cm^{-3}$$

2) μ_h?

$$\mu_h = 240\ cm^2/V.s \Rightarrow D_h = \frac{KT}{q}\mu_h = \frac{26\,m\cancel{eV}}{1e} \cdot 240\ \frac{cm^2}{\cancel{V}s} = 6.2\ cm^2/s$$

$$\mu_e = 530\ cm^2/V.s \Rightarrow D_e = 13.8\ cm^2/s$$